Baryonium, Tetra-quark State and Glue-ball in Large N_c QCD

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(Dated: April 5, 2017)

Abstract

From the large-N_c QCD point of view, baryonia, tetra-quark states, hybrids, and glueballs are studied. The existence of these states is argued for. They are constructed from baryons. In $N_f = 1$ large N_c QCD, a baryonium is always identical to a glueball with N_c valence gluons. The ground state 0^{-+} glueball has a mass about 2450 MeV. $f_0(1710)$ is identified as the lowest 0^{++} glueball. The lowest four-quark nonet should be $f_0(1370)$, $a_0(1450)$, $K_0^*(1430)$ and $f_0(1500)$. Combining with the heavy quark effective theory, spectra of heavy baryonia and heavy tetra-quark states are predicted. $1/N_c$ corrections are discussed.

PACS numbers: 12.39.Mk, 14.20.-c, 12.38.Lg

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I. INTRODUCTION

The large-N_c limit is one of the most important methods of non-perturbative QCD [1, 2, 3]. Properties of mesons can be observed from the analysis of planar diagrams and baryons from the Hartree-Fock picture. While mesons are free, the interaction between baryons is strong which scales as $\mathcal{O}(N_c)$. This might imply a duality between the meson and baryon sectors [3]. This implication comes from the similarity between large-N_c baryons and Polyakov-'t Hooft monopoles by taking $1/N_c$ as a coupling constant. This duality was indeed partially realized [4] in the Skyrme model [5] in which a baryon is regarded as a soliton of a meson theory.

In this work, hadron spectra are described starting from baryons. Besides mesons and baryons, certain multiquark systems are also included. Generally, color-singlet multiquark-gluon systems have been expected naively due to their colorlessness. Now we try to give deeper reasoning about their existence and their description from the large-N_c QCD point of view. In principle, for large-N_c QCD, baryon theories exist as dual ones to meson theories. Such theories, however, are strongly interacting, which are lack of a perturbative description. Taking baryons as the starting point can be traced back to Fermi and Yang [6] long before the establishment of QCD, while strange baryons were included by Sakita [7]. Its recent version can be found in Ref. [8]. Alternative to taking baryons as basic building blocks, we will still use the Hartree-Fock picture of large-N_c baryons as Witten did [3] to do semi-quantitative analyses.

Baryons themselves have been studied in Refs. [3, 9, 10]. There are N_c valence quarks in a baryon. Baryon-baryon interactions are strong. Molecular states of baryons can exist due to their strong interactions. And they are just the nuclei [3].

II. BARYONIA

Let us consider baryon-antibaryon systems. The baryon-antibaryon system was mentioned in Ref. [3]. We will study its properties by assuming that it forms a bound state. The interaction between a baryon and an antibaryon can be as strong as that of baryon-baryon systems. Therefore, we expect that molecular states of a baryon and an antibaryon also exist. Because of baryon-antibaryon annihilation, the baryon and anti-baryon in a
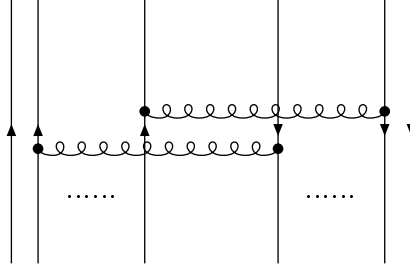
FIG. 1: Interaction inside a baryonium

baryonium is attractive at small distances, baryonia are more deeply bounded than nuclei.

The relevant interactions can be classified into two cases. The first one is that of glueball exchanges. The antibaryon inside a baryonium is not necessarily the anti-particle of the baryon in the baryonium. A possible example is the baryonium composed of $\Delta^{++}(uuu)$ and $\bar{\Delta}^-(\bar{d}\bar{d}\bar{d})$ where the valence quark contents are given. In this case, the interaction inside a baryonium is described by Fig. 1 when the two baryons are close enough. Its N_c-dependence can be seen from the following. Each gluon-quark vertex contributes $1/\sqrt{N_c}$. There are N_c^2 possible ways to make the first gluon. To make the second gluon, however, there are only N_c possible ways, because the two gluons must compose a color singlet state. Therefore, the interaction energy is still proportional to N_c. Generally a baryonium mass is about $2N_c\Lambda_{\rm QCD}$. The binding energy, though proportional to N_c, is expected to be smaller than baryon masses $\sim N_c\Lambda_{\rm QCD}$ because baryons are color singlet. This is consistent with the molecular picture of baryonia. In terms of the hadron language, the interaction is mediated by glueballs with the glueball-baryon coupling $\sim \sqrt{N_c}$. Because of heaviness of glueballs, such a t-channel glueball exchange interaction might be suppressed unless it happens at small distances. The short range interaction is also required by the confinement.

The second case is that of meson exchanges. When a quark and an antiquark have a common flavor in the baryonium, the interaction given by Fig. 2 [3] plays a role. This interaction is regarded as a meson exchange. Once any of the quarks is able to annihilate with any of the antiquarks, namely, all the quarks have the same flavor in the baryonium, Fig. 2 is of equal N_c importance as Fig. 1. The realistic situation is an interplay of the two
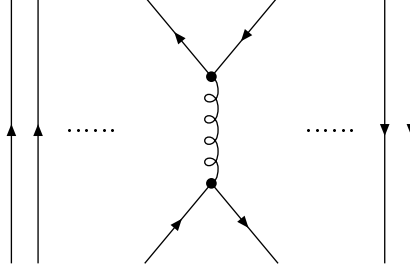
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FIG. 2: Interaction inside a baryonium with a quark pair annihilation [3]

kinds of interactions described in Figs. 1 and 2. The amplitude of the baryon-antibaryon scattering described in Fig. 2 by assuming $N_f = 1$ [3] is proportional to N_c. This divergent large N_c behavior of the amplitude matches with that of baryon kinetic energies, and shows the strong interacting behavior between the baryon and the antibaryon.

Assuming the existence of baryonia, interesting observation about hybrids and glueballs can be seen in the large N_c limit. In the case that all the quarks have the same flavor in a baryonium ($N_f = 1$), a hybrid state with valence content of $(N_c - 1)$ quarks, $(N_c - 1)$ antiquarks and one gluon, for instance, is large-N_c enhanced due to the same reason of that baryon interaction is strong, as can be seen by cutting Fig. 2 in the middle. This state and the baryonium transfer into each other constantly by the strong dynamics. Therefore in fact, this hybrid state is physically not different from the baryonium in the large-N_c limit. For the same reason, this state can be equally identified as being composed of $(N_c - 2)$ quarks, $(N_c - 2)$ antiquarks and two gluons. Furthermore, and remarkably, a glueball composed of N_c valence gluons is also in fact indistinguishable from the baryonium. Referring to Fig. 2, it is easily seen that a valence gluon has a mass of 2 valence quarks, which is about $2\Lambda_{\mathrm{QCD}}$. The glueball composed of two valence gluons therefore has a mass about $4\Lambda_{\mathrm{QCD}}$. It is clear that the glueball with N_c valence gluons has a mass of the baryonium. We have stated that the baryonium, the hybrid and the glueball are the same state in $N_c = \infty$, $N_f = 1$ QCD. Although at the quark-gluon level this is not true, there is no way to distinguish them at the hadron level due to the confinement.

On the other hand, above reasoning also implies that the existence of glueballs supports
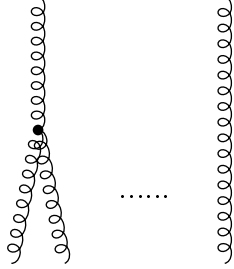
FIG. 3: N_c valence gluons transfer to $N_c - 1$ valence gluons

the existence of baryonia. In the $N_f = 0$ case, the confinement requires the existence of glueballs as hadrons. Light glueballs are massive with masses several times of Λ_{QCD}. Adding a single flavor into this case, lowest new hadrons include η' meson due to chiral symmetry spontaneous breaking and anomaly, and Δ^{++} baryon. In the case of $N_c \to \infty$, glueballs and baryonia ($\Delta^{++}, \bar{\Delta}^{--}$) with same quantum numbers are indistinguishable. Baryonia are then generally expected. The constituent quark mass is determined to be half of the constituent gluon mass. With more flavor added, many new baryons with various flavor quantum numbers appear. The baryonium existence beyond $N_f = 1$ is less sound than the case of $N_f = 1$.

Note a glueball of N_c valence gluons can transfer to a glueball of $(N_c - 1)$ valence gluons. But this transition rate is $\mathcal{O}(1)$. This is seen from Fig. 3. The three-gluon vertex has a factor of $1/\sqrt{N_c}$. There are N_c ways to make the transition. By considering the color quantum numbers of the gluons in Fig. 3, we know that the N_c ways do not add coherently. The rate of Fig. 3 is $1/N_c$. Then the total transition rate from a N_c valence gluon state into a $(N_c - 1)$ valence gluon state is $\mathcal{O}(1)$. It is $1/N_c$ suppressed compared to the transition rate of a baryonium into a one-gluon hybrid state. Therefore the N_c valence gluon state distinguishes itself from the $(N_c - 1)$ valence gluon state. The two states have $\mathcal{O}(1)$ mixing in amplitudes. In other words, the number of valence gluons inside a glueball is well-defined in the large N_c limit.

III. DIQUARK-ANTIDIQUARKS

Unlike Ref. [3], we shall distinguish baryonium states and diquark-antidiquark states. Taking a color singlet quark pair away from a baryonium, a color singlet $(N_c - 1)$-quark-$(N_c - 1)$-antiquark state can be always formed. Their existence has been argued for in Ref. [3]. The $(N_c - 1)$ quarks form a $\bar{\mathbf{N}}_{\mathbf{c}}$ representation, and $(N_c - 1)$ antiquarks a $\mathbf{N}_{\mathbf{c}}$ representation. Such a state has a mass of about $2(N_c - 1)\Lambda_{\text{QCD}}$. It is large N_c extension of the tetra-quark state, by taking $N_c = 3$, this state is the diquark-antidiquark one. In such a $(N_c - 1)$-quark-$(N_c - 1)$-antiquark system, if all the quark flavors are the same, the hybrid state of $(N_c - 2)$-quark, $(N_c - 2)$-antiquark and one gluon is large-N_c enhanced and is physically not different from the $(N_c - 1)$-quark-$(N_c - 1)$-antiquark system.

Let us consider in more detail the (N_c-1)-quark-(N_c-1)-antiquark state. Taking $N_f = 1$, processes similar to that described in Fig. 2 happen. In most of the cases when the gluon is formed from quarks with different colors, interaction keeps the $(N_c - 1)$ quarks in $\bar{\mathbf{N}}_{\mathbf{c}}$ representation and $(N_c - 1)$ antiquarks in $\mathbf{N}_{\mathbf{c}}$ representation. This process amplitude is proportional to N_c. When the gluon is formed from quarks with the same color, the final $(N_c - 1)$ quarks generally do not stay in $\bar{\mathbf{N}}_{\mathbf{c}}$ representation. But this transition amplitude is $\mathcal{O}(1)$. The similar result can be obtained if we consider t-channel gluon exchanges. Therefore, quark configuration of a state with the $(N_c - 1)$ quarks in $\bar{\mathbf{N}}_{\mathbf{c}}$ representation and $(N_c - 1)$ antiquarks in $\mathbf{N}_{\mathbf{c}}$ representation makes sense in the large N_c limit. In real situation $(N_c = 3)$, this is to say that $\mathbf{3} \otimes \bar{\mathbf{3}}$ tetraquark configuration does not mix with $\mathbf{6} \otimes \bar{\mathbf{6}}$ tetraquark configuration when $N_c = 3$ is considered to be large.

Further taking a color singlet quark pair away, a $(N_c - 2)$-quark-$(N_c - 2)$-antiquark state is then formed with a mass being about $2(N_c-2)\Lambda_{\text{QCD}}$. The (N_c-2) quarks form a $\bar{\mathbf{N}}_{\mathbf{c}} \otimes \bar{\mathbf{N}}_{\mathbf{c}}$ representation, and $(N_c - 2)$ antiquarks a $\mathbf{N}_{\mathbf{c}} \otimes \mathbf{N}_{\mathbf{c}}$ representation. The situation is more complicated.

The above procedure might continue. Finally, a valence quark-antiquark state is formed with a mass being about $2\Lambda_{\text{QCD}}$, which is just a meson. (Note that chiral symmetry breaking cannot be counted in this framework.)

In the same manner as we have discussed for the baryonium and the glueball in the last section, the single flavor (N_c-1)-quark and (N_c-1)-antiquark system is not distinguishable from the glueball composed of $N_c - 1$ valence gluons. And the two-quark and two-antiquark

system is indistinguishable from the glueball composed of two valence gluons.

Once more flavors are included, above large N_c consideration becomes more complicated. As an example, the $(N_c - 1)$ quarks stay in the lowest energy state if they take $(N_c - 1)$ different flavors. In this case, the $(N_c - 1)$ quark - $(N_c - 1)$ antiquark state transition to the hybrid state of $(N_c - 2)$ quark, $(N_c - 2)$ antiquark and one gluon is $1/N_c$ suppressed compared to the single flavor case. They cannot be the same state in the large-N_c limit, but they have $\mathcal{O}(1)$ mixing in amplitudes.

IV. DECAYS AND BINDING ENERGIES

Decays of baryoniums and $(N_c - 1)$ quark-$(N_c - 1)$ antiquark states have been discussed in Ref. [3]. A baryonium decays into one meson and a $(N_c - 1)$ quark-$(N_c - 1)$ antiquark state. A $(N_c - 1)$ quark-$(N_c - 1)$ antiquark state decays into one meson and a $(N_c - 2)$ quark-$(N_c - 2)$ antiquark state. Such cascade decays continue until the final 4-quark state decays into two mesons. These decays are slow.

These decay rates are $\mathcal{O}(1)$. This is easy to see from the fact that a color-singlet quark pair drops out of a baryonium or of a $(N_c - 1)$ quark-$(N_c - 1)$ antiquark state with an amplitude of $\mathcal{O}(1)$.

Theoretically, baryon-antibaryon systems might be difficult to deal with, because the typical energy of such interaction is proportional to N_c [3] which is the same N_c-dependence of baryon masses $\sim N_c \Lambda_{\mathrm{QCD}}$. However, as we have argued in Sect. II, it is expected that the baryon-antibaryon interacting energy is smaller than $N_c \Lambda_{\mathrm{QCD}}$ due to the confinement. Furthermore, it is expected from large-N_c QCD that the baryon-baryon typical interacting energy is also proportional to N_c and phenomenologically, nuclear physics shows that the typical binding energy of a baryon inside a nuclear is only about a few MeV. Therefore, we expect that the binding energy of a baryonium is actually a lot smaller than a baryon mass. It makes molecular description (in the QCD sense) of baryonium states meaningful. Such a system in fact can be well described by non-relativistic quantum mechanics. Consequently, baryon spins decouple from the dynamics of baryoniums.

To express the problem more clearly, in the large N_c-limit, baryon-baryon binding energy is $N_c \lambda$, and baryon-antibaryon binding energy is $N_c \lambda'$. The confinement argument gives that $\lambda < \Lambda_{\mathrm{QCD}}$ and $\lambda' < \Lambda_{\mathrm{QCD}}$.

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The slow transition of a baryonium to a $(N_c - 1)$ quark-$(N_c - 1)$ antiquark state and one meson implies that the interacting strength inside the baryonium and the $(N_c - 1)$ quark-$(N_c - 1)$ antiquark state are the same. In other words, in the large N_c limit the constituent quark mass (Λ_{QCD}) can be taken the same in these two kinds of hadrons.

V. ANALYSIS

We will make a numerical illustration through analyzing realistic situation. In the semi-quantitative analysis, we only consider ground state hadrons. Being accurate requires specification of meaning of Λ_{QCD} we have used. This quantity describes the energy of an individual quark inside baryons. In the following, we define $\bar{\Lambda}_{\mathrm{QCD}}$ to replace Λ_{QCD},

$$M_{\text{ground state baryon}} \equiv N_c \bar{\Lambda}_{\mathrm{QCD}} , \tag{1}$$

namely $\bar{\Lambda}_{\mathrm{QCD}}$ is identified as a constituent quark mass in baryons. Taking $N_c = 3$, we have $\bar{\Lambda}_{\mathrm{QCD}} = (362 \pm 50)$ MeV by taking the average of masses of a nucleon and Δ^{++}.

The analysis depends crucially on how much the baryonium binding energy is. Different binding energy corresponds to different physical picture of hadrons. As we will see it determines which group of hadron particles in the Particle Date Book is identified as 4-quark states. We will mainly take a 10 MeV binding energy. The error of this 10 MeV binding energy is hard to estimate. Some other phenomenological works [12] use about 300 MeV binding energy. A significantly larger binding energy will be considered briefly as a comparison later.

The numerical analysis would be more appropriate to the $N_f = 1$ case, however, it will go beyond that to $N_f = 2$ and 3 without further mention of that the latter cases are more assumption-dependent.

A. 10 MeV binding energy

We take the binding energy to be 10 MeV, that means that $\lambda' \simeq \lambda$ or a little bit larger, considering both λ and λ' are essentially determined by Λ_{QCD} and the confinement. Nuclear physics tells us that $N_c \lambda$ is about a few MeV. We expect that $N_c \lambda' \simeq 10$ MeV typically . A recent study from the Skyrmeon model shows that the baryonium binding energy is indeed about 10 MeV [11].

Consider the case of only one flavor, the lowest baryonium is s-wave $(\Delta^{++}\ \bar{\Delta}^{--})$, with the quark being the up-quark as an example, its mass is about $2M_{\Delta^{++}} - 10\text{MeV} \simeq 2450$ MeV. As we have argued, this state can be also identified as a 0^{-+} ground state of three-gluon glueball in the large N_c limit. The infered constituent gluon mass is consistent with those via other methods [13]. The actual glueball mass maybe a bit lower than the above value, because the actual flavor number is more than one. With one more light flavor being introduced, numerically it is estimated that the mass of the lowest proton-antiproton molecular state is about $2M_N - 10$ MeV $\simeq 1866$ MeV. This molecular state mixes with the 0^{-+} glueball. In the large N_c limit, this mixing depends on large N_c generalization of the nucleon, which we do not consider in this paper.

Experimentally, BES collaboration has found two baryonium candidates, $X(1860)$ [14] and $X(1835)$ [15]. They were then theoretically studied [11, 16]. Considering the uncertainties of the binding energy, these states are consistent with our expectation. Furthermore, the corresponding 0^{-+} state exists due to the different baryon spin combination. Their approximate degeneracy is a result of baryon spin decoupling. As a check, $p\bar{n}$, $n\bar{p}$ and $n\bar{n}$ states should have a degenerate mass as $p\bar{p}$ which is about 1835 MeV or 1860 MeV. In the three light flavor case, the lowest baryonium $(p, \bar{\Lambda})$ or $(n, \bar{\Lambda})$ is expected to have a mass of $M_N + M_\Lambda - 10$ MeV $\simeq 2045$ MeV and the baryonium $(\Lambda, \bar{\Lambda})$ with a mass of $2M_\Lambda - 10\text{MeV} \simeq 2220$ MeV. $(p, \bar{\Lambda})$ is consistent with the experiment $\sim 2075 \pm 13$ MeV [17].

A $(N_c - 1)$ quark-$(N_c - 1)$ antiquark state is of a mass about $2(N_c - 1)\bar{\Lambda}_{\text{QCD}}$. From the argument of last section, it is reasonable to take the constituent quark mass in a tetra-quark state to be the same as that in a baryonium. In the case of only one flavor, the 4-quark state is $(uu\ \bar{u}\bar{u})$. Its lowest mass is estimated to be $(2M_{\Delta^{++}} - 10\text{MeV}) - 2\bar{\Lambda}_{\text{QCD}} \simeq 1740 \pm 100$ MeV. This state can be also regarded as a $(u\bar{u}g)$ hybrid or a 0^{++} ground state two-gluon glueball. It should be identified as $f_0(1710)$ in our scheme. Our estimation is consistent with lattice calculation [18].

With one more flavor included, the lowest mass can be written as

$$2(N_c - 1)\bar{\Lambda}_{\text{QCD}} \simeq [M_{X(1835)} \text{ or } M_{X(1860)}] - 2\bar{\Lambda}_{\text{QCD}} \simeq (1110 - 1140) \pm 100\text{MeV}. \quad (2)$$

This is the 4-quark ground state $(ud\ \bar{u}\bar{d})$ with both spin and isospin 0. It can be identified as $f_0(1370)$ which has a mass ranging from 1200 to 1500 MeV [19]. As being noted, it mixes with the hybrid state $(q\bar{q}g)$ with q standing for the u- or d-quark.

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As we have seen that 4-quark state mass estimation has an uncertainty ~ 200 MeV, above numbers are of limited use practically. Our point is that within the uncertainty, there should be 4-quark states, and they indeed have experimental correspondence. After the strange quark is introduced, three kinds of lowest diquarks can be formed, ud, us and ds. The 4-quark states form a nonet. They were studied by many authors [20, 21, 22, 23, 24]. In our work, their mass differences are expected to be determined by the strange quark mass, which do not subject to the large uncertainty of large N_c approximation. So the ground 4-quark states are naturally identified as $f_0(1370)$, $a_0(1450)$, $K_0^*(1430)$, $f_0(1500)$; more explicitly, $f_0(1370)(ud\bar{u}\bar{d})$, $K_0^*(1430)(ud\bar{u}\bar{s}, ud\bar{d}\bar{s}, us\bar{u}\bar{d}, ds\bar{u}\bar{d})$, $a_0(1450)(us\bar{d}\bar{s}, ds\bar{u}\bar{s})$, $a_0(1450)(s(n\bar{n})_-\bar{s})$, $f_0(1500)(s(n\bar{n})_+\bar{s})$, where $(n\bar{n})_{\pm} \equiv (u\bar{u} \pm d\bar{d})/\sqrt{2}$. The mixing among the f_0 states are an $\mathcal{O}(1)$ effect. Note that our 4-quark state identification is different from most of previous studies [20, 21, 22, 23] where it is σ, κ, $a_0(980)$ and $f_0(980)$ that are taken to be the lowest 0^{++} 4-quark states.

B. Large binding energy

As a comparison, let us consider the case of large baryon-antibaryon interacting energies. We know that $\lambda \simeq \lambda'$ is still an assumption. If the annihilation effect is important, λ' is possibly a lot larger. Refs. [12] took it to be about 200 MeV which still makes the molecular picture of baryoniums sensible. Now we fix $N_c\lambda'$ by requiring that 4-quark ground states correspond to σ, κ, $a_0(980)$ and $f_0(980)$. For a large binding energy, the mass of a diquark-antidiquarks is written as $2(N_c - 1)\bar{\Lambda}''_{\text{QCD}}$, while the mass of a baryonium is $2N_c\bar{\Lambda}'_{\text{QCD}}$. It is reasonable for ground states that $\bar{\Lambda}''_{\text{QCD}} \simeq \bar{\Lambda}'_{\text{QCD}}$ in the large-N_c limit, because we can imagine a dynamical $\mathcal{O}(1)$ process to generate a ground state diquark-antidiquark from a ground state baryonium via emitting a meson with the mass being $2\bar{\Lambda}_{\text{QCD}}$. The interaction energy between a diquark-antidiquark and a meson is $1/N_c$ suppressed compared to $N_c\lambda'$, and will be neglected, as we have also implicitly done in last subsection. Taking the strange quark mass $m_s = 150$ MeV, from $M_{f_0(980)} \simeq 2(N_c-1)\bar{\Lambda}'_{\text{QCD}}+2m_s$, we obtain that $\bar{\Lambda}'_{\text{QCD}} \simeq 170$ MeV. In this case, $M_\sigma \simeq 680$ MeV. Lowest 0^{-+} baryoniums should be then about 1020 MeV which means a 960 MeV binding energy. However, there is no such baryonium correspondence in the Particle Data Book actually.

Therefore, considering practical situation, large-N_c QCD analyses prefer a small baryo-

nium binding energy. In the following we will not consider the large binding energy case.

C. Decay widths

A baryonium decays into one meson and one tetra-quark state, and a tetra-quark state into two mesons [3]. The decay rates are $\mathcal{O}(1)$. Therefore, from the large N_c point of view, diquark-antidiquark states decay slowly, and this also makes them distinguishable from two meson states. The 0^{-+} baryonium $X(1835)$ or $X(1860)$ decays in p-wave into $f_0(1370)$ and σ. Note that it cannot decay to s-wave $f_0(1370)$ and η' due to the phase space. Therefore the dominant decay products are $\rho\rho\pi\pi$. This can be checked by future experiments.

VI. HEAVY HADRONS

Now we consider the case of inclusion of a single heavy quark. Heavy quark effective theory (HQET) [25] provides a systematic way to investigate hadrons containing a single heavy quark. It is an effective field theory of QCD for such heavy hadrons. In the limit $m_Q/\Lambda_{\text{QCD}} \to \infty$, the heavy quark spin-flavor symmetry is explicit. The hadron mass is expanded as

$$M_H = m_Q + \bar{\Lambda}_H + \mathcal{O}(1/m_Q) . \tag{3}$$

To obtain the HQET defined, universal heavy hadron mass $\bar{\Lambda}_H$, however, some non-perturbative QCD methods have to be used. We can apply the large N_c method. Heavy baryons were studied via this method [10, 26, 27, 28, 29]. Let us first consider the relation of the quantity $\bar{\Lambda}_H$ of a ground state heavy baryon and the nucleon mass. Heavy baryons contain $(N_c - 1)$ light quarks, and one "massless" heavy quark (modular m_Q). The mass or the energy of the baryon is determined by the summation of the energies of individual quarks. The kinetic energy of the heavy quark is typically $\bar{\Lambda}_{\text{QCD}}$ like that of the light quark. The interaction energy between the heavy quark and any of the light quarks is typically $\bar{\Lambda}_{\text{QCD}}/\text{N}_c$. So the interaction energy between the heavy quark and the whole light quark system scales as $\bar{\Lambda}_{\text{QCD}}$. However, the total interaction energy of the light quark system itself scales as $N_c\bar{\Lambda}_{\text{QCD}}$. Therefore in the large N_c limit, $\bar{\Lambda}_H = M_N$ where the uncertainty of the equation is $\mathcal{O}(1) \sim \bar{\Lambda}_{\text{QCD}}$ in $1/N_c$ expansion.

Actually in the mass relation between heavy baryons and corresponding light baryons

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under the large N_c limit, the uncertainty is smaller than $\bar{\Lambda}_{\rm QCD}$. This is because the heavy quark constituent mass (modular m_Q) does not deviate from $\bar{\Lambda}_{\rm QCD}$ very much. For an example, the Λ_Q baryon mass $\bar{\Lambda}_{\Lambda_Q}$ is about 0.80 GeV [30]. It is more reasonable to take $M_N - \bar{\Lambda}_{\Lambda_Q} \sim 0.15$ GeV to be the uncertainty in the following analysis.

Heavy baryoniums containing a heavy quark are analyzed in the same large-N_c spirit of the light quark case. The 0^- ground state baryoniums $(\Lambda_c, \bar{N})$ and $(\Lambda_c, \bar{\Lambda})$ have masses

$$
\begin{aligned}
M_{(\Lambda_c, \bar{N})} &\simeq m_c - m_s + M_{(N, \bar{\Lambda})} \simeq 3.33 \pm 0.15 \text{ GeV}, \\
M_{(\Lambda_c, \bar{\Lambda})} &\simeq m_c - m_s + M_{(\Lambda, \bar{\Lambda})} \simeq 3.50 \pm 0.15 \text{ GeV},
\end{aligned}
\tag{4}
$$

where m_c is taken to be 1.43 GeV [30]. This is consistent with naive estimation $M_{(\Lambda_c, \bar{N})} = M_{\Lambda_c} + M_N - 10$ MeV $\simeq 3.21$ GeV and $M_{(\Lambda_c, \bar{\Lambda})} = M_{\Lambda_c} + M_\Lambda - 10$ MeV $\simeq 3.40$ GeV. They also have corresponding degenerate 1^- states due to baryon spin decoupling.

For heavy diquarks, because of the heavy quark spin symmetry, existence of a spin-zero diquark Qq implies that a Qq spin-one diquark also exists. This point was also noticed in Ref. [23]. The spectrum of the lowest 4-quark charm states is

$$
\begin{aligned}
M_{(cd, \bar{u}\bar{d})} &= M_{(cu, \bar{u}\bar{d})} = m_c + M_{f_0(1370)} \simeq (2.54 - 2.57) \pm 0.15 \text{ GeV}, \\
M_{(cd, \bar{u}\bar{s})} &= M_{(cu, \bar{u}\bar{s})} = M_{(cd, \bar{d}\bar{s})} = M_{(cu, \bar{d}\bar{s})} = M_{(cs, \bar{u}\bar{d})} \\
&= m_c + M_{a_0(1450)} \simeq 2.84 \pm 0.15 \text{ GeV}, \\
M_{(cs, \bar{u}\bar{s})} &= M_{(cs, \bar{d}\bar{s})} = m_c + M_{f_0(1500)} \simeq 2.94 \pm 0.15 \text{ GeV}.
\end{aligned}
\tag{5}
$$

In the heavy quark limit, we have the degeneracy of 0^+, 1^+ and 2^+ 4-quark states.

Therefore, we expect a rich charm hadron spectrum ranging from 2.54 GeV to 3.50 GeV. The $1/m_Q$ uncertainty is about $\Lambda_{\rm QCD}^2/m_c \sim 60$ MeV. The bottom case is the same except for smaller $1/m_Q$ correction because of the heavy quark flavor symmetry,

$$
\begin{aligned}
M_{(\Lambda_b, \bar{N})} &\simeq m_b - m_s + M_{(\Lambda, \bar{N})} \simeq 6.73 \pm 0.15 \text{ GeV}, \\
M_{(\Lambda_b, \bar{\Lambda})} &\simeq m_b - m_s + M_{(\Lambda, \bar{\Lambda})} \simeq 6.90 \pm 0.15 \text{ GeV}, \\
M_{(bd, \bar{u}\bar{d})} &= M_{(bu, \bar{u}\bar{d})} \simeq m_b + M_{f_0(1370)} \simeq (5.94 - 5.97) \pm 0.15 \text{ GeV}, \\
M_{(bd, \bar{u}\bar{s})} &= M_{(bu, \bar{u}\bar{s})} = M_{(bd, \bar{d}\bar{s})} = M_{(bu, \bar{d}\bar{s})} = M_{(bs, \bar{u}\bar{d})} \\
&\simeq m_b + M_{a_0(1450)} \simeq 6.24 \pm 0.15 \text{ GeV}, \\
M_{(bs, \bar{u}\bar{s})} &= M_{(bs, \bar{d}\bar{s})} \simeq m_b + M_{f_0(1500)} \simeq 6.34 \pm 0.15 \text{ GeV},
\end{aligned}
\tag{6}
$$

where $m_b \simeq 4.83$ GeV [30]. $M_{(\Lambda_b, \bar{N})}$ and $M_{(\Lambda_b, \bar{\Lambda})}$ are consistent with $M_{\Lambda_b} + M_n - 10$ MeV $\simeq$ 6.61 GeV and $M_{\Lambda_b} + M_\Lambda - 10$ MeV $\simeq 6.80$ GeV, respectively.

For hadrons containing a pair of heavy quarks, the two heavy quarks intend to combine into a tighter object which is described by non-relativistic QCD. However, if the two heavy quarks are separated by $1/\Lambda_{\mathrm{QCD}}$ or more in certain hadrons, the above HQET procedure can be applied to these hadrons. For examples, the following lowest baryoniums and 4-quark states may exist,

$$
\begin{aligned}
M_{(\Lambda_c, \bar{\Lambda}_c)} &\simeq 2m_c - 2m_s + M_{(\Lambda, \bar{\Lambda})} \simeq 4.78 \pm 0.15 \ \mathrm{GeV}\,, \\
M_{(cu, \bar{c}\bar{u})} &= M_{(cu, \bar{c}\bar{d})} = M_{(cd, \bar{c}\bar{u})} = M_{(cd, \bar{c}\bar{d})} \simeq 2m_c + M_{f_0(1370)} \simeq (3.97 - 4.00) \pm 0.15 \ \mathrm{GeV}\,, \\
M_{(cu, \bar{c}\bar{s})} &= M_{(cd, \bar{c}\bar{s})} = M_{(cs, \bar{c}\bar{u})} = M_{(cs, \bar{c}\bar{d})} \simeq 2m_c + M_{a_0(1450)} \simeq 4.27 \pm 0.15 \ \mathrm{GeV}\,, \\
M_{(cs, \bar{c}\bar{s})} &\simeq 2m_c + M_{f_0(1500)} \simeq 4.37 \pm 0.15 \ \mathrm{GeV}\,,
\end{aligned}
\tag{7}
$$

where $M_{(\Lambda_c, \bar{\Lambda}_c)}$ is consistent with $2M_{\Lambda_c} - 10 \ \mathrm{MeV} \simeq 4.83 \ \mathrm{GeV}$, and the uncertainty due to $1/m_Q$ effects is $\Lambda_{\mathrm{QCD}}^2/2m_c \simeq 30 \ \mathrm{MeV}$. The state $(cs, \bar{c}\bar{s})$ is consistent with $Y(4260)$ [22] which cannot be identified as $(\Lambda_c, \bar{\Lambda}_c)$ [31] in our scheme. Considering $1/N_c$ uncertainties, $X(3940)$ can be $(cq\, \bar{c}\bar{q})$ $(q = u, d)$. In that case, charged $(cu\, \bar{c}\bar{d})$ state is also expected around 3940 MeV.

VII. SUMMARY AND DISCUSSION

From the large N_c QCD point of view, we have considered baryoniums, four-quark states, hybrids and glueballs. The existence of baryonium states is argued for from existence of nuclei. These hadrons are constructed from baryons. We have argued that in $N_f = 1$ large N_c QCD, a baryonium is always identical to a glueball with N_c valence gluons. $f_0(1370)$, $a_0(1450)$, $K_0^*(1430)$ and $f_0(1500)$ are identified as the lowest four-quark nonet. The glueball with three valence gluons has a mass about 2450 MeV. $f_0(1710)$ is identified as the glueball with two valence gluons. Combining with HQET, we have predicted heavy baryoniums and heavy four-quark states.

This work can be viewed as a large N_c QCD extension of the Fermi-Yang model. A classification of hadrons is given a large N_c QCD basis. We have constructed hadron spectra from baryoniums, because our starting point is baryons. The reversed procedure is not necessarily true. For an example, the one-gluon hybrid state existing in this scheme can always be generated from or identified as a diquark-antidiquark state. The large N_c QCD arguments and consequent estimation help us understanding relevant experimental results.

However, they do not result in a precise mathematical description for the hadrons we have studied in this paper. Finding such a systematic description is one of the task in sloving nonperturbative QCD.

The uncertainties of the analysis should be discussed. Of course, existence of baryoniums as well as the 10 MeV binding energy is still an assumption, but it is supported by the large N_c analysis. Even if $N_c = 3$, baryoniums are still expected. In many cases, qualitative conclusion of large N_c QCD is also true when $N_c = 3$. The baryon-baryon strong interaction in large N_c QCD implies existence of baryon bound states. Indeed in real QCD with $N_c = 3$, nuclei exist. The meson-meson interaction is vanishing in the large N_c-limit, therefore, there is no molecular states of mesons. And in the real world, meson molecular states seem non-existent.

It is important to discuss the $1/N_c$ corrections to our numerical analysis. The estimated masses of the baryoniums and diquark-antidiquark states would have $\mathcal{O}(1) \sim (200 - 300)$ MeV uncertainties. But the relative masses of the above hadrons have no that large uncertainties. For an example, once the binding energy of $p\bar{p}$ is fixed as 10 MeV, then the $\Lambda\bar{\Lambda}$ binding energy is 10 MeV with an uncertainty of about 30% due to SU(3) violation. Namely, the baryoniums $p\bar{p}$ and $\Lambda\bar{\Lambda}$ mass difference does not subject to large $1/N_c$ corrections. More accurate treatment of baryoniums can be similar to that in Refs. [9, 10] by taking account baryonium binding energies. Furthermore, for the diquark-antidiquark states, their mass differences similarily have no large $1/N_c$ uncertainties. For another example, mass differences of states $(cd, \bar{u}\bar{d})$, $(cd, \bar{u}\bar{s})$ and $(cs, \bar{u}\bar{s})$ do not subject to $\bar{\Lambda}_{\text{QCD}}$ uncertainty. The discovery of the baryoniums and the diquark-antidiquark states with a single heavy quark and their mass estimation given in Eqs. (3-5) will be tests of our understanding in the near future.

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